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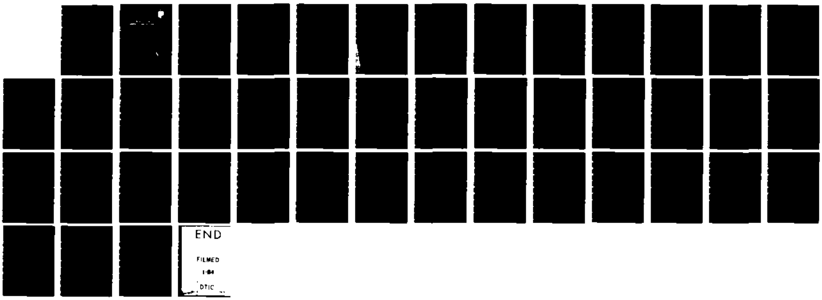

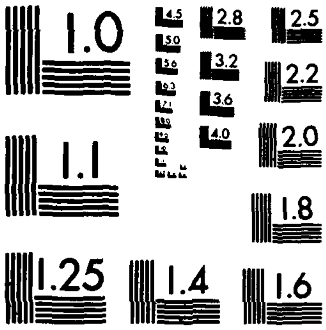

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July 1983

EFFECTS OF MUTUAL COUPLING ON THE PERFORMANCE OF ADAPTIVE ARRAYS

The Ohio State University

Inder J. Gupta and Aharon A. Ksienski

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
The effect of mutual coupling between array elements on the performance of adaptive arrays is examined. The study includes both steady state and transient performance. An expression for the steady state output signal-to-interference-plus-noise ratio (SINR) of adaptive arrays, taking into account the mutual coupling between the array elements, is derived. The expression is used to assess the steady state performance of adaptive arrays. The transient response is studied by computing the eigenvalues

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) associated with the signal covariance matrix. The optimum excitation to maximize the output SINR of Applebaum type adaptive arrays in the presence of mutual coupling is also given.

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I. INTRODUCTION

It has been shown [1] that the performance of an adaptive antenna array is strongly affected by the electromagnetic characteristics of the antenna array. An important electromagnetic characteristic of an antenna array is the mutual coupling between its elements. In the above work, mutual coupling between the antenna elements was, however, ignored, i.e., the antenna elements were assumed to be isolated from each other. In practice, elements of an antenna array have mutual coupling, which in turn affect the gain, beamwidth, etc., of the array. Mutual coupling becomes particularly significant as the interelement spacing is decreased.

In this report, the effect of mutual coupling on the performance of adaptive arrays is studied. It is shown that the mutual coupling does affect the performance of adaptive arrays and these effects are significant even for large interelement spacings, i.e., for spacing of more than half a wavelength. The effect is rather drastic as the interelement spacing drops below a half wavelength. In fact, for a fixed aperture with half wavelength spaced elements, the introduction of additional elements can degrade the array performance. The failure to recognize the presence of mutual coupling will degrade the performance of Applebaum type adaptive arrays more than that of LMS arrays since the optimum excitation has to be modified both in phase and amplitude to include the changes in the desired signal vector due to the presence of mutual coupling.

In section II, an analytic expression for the steady state output signal-to-interference-plus-noise ratio (SINR) of an adaptive array is derived. The expression takes into account the mutual coupling between the array elements and involves the normalized impedance matrix of the array elements. The expression is used to study the effect of mutual coupling on the performance of adaptive arrays and it is shown that the output SINR of the array depends upon the mutual coupling between its elements. In section III, the effect of mutual coupling on the transient performance of adaptive arrays is studied. It is shown that the presence of mutual coupling between the array elements reduces the speed of response of an adaptive array. In section IV, the optimum excitation to maximize the output SINR of Applebaum type adaptive arrays in the presence of mutual coupling is found. Section V contains our conclusions.

II. STEADY STATE PERFORMANCE OF AN ADAPTIVE ARRAY IN THE PRESENCE OF MUTUAL COUPLING

The output SINR of an adaptive array is the most commonly accepted measure of its steady state performance, and will accordingly be derived first. The expression takes into account the mutual coupling between the array elements. The expression will be used to study the effect of mutual coupling on the performance of various adaptive arrays.

The basic diagram of an adaptive array is shown in Figure 1. The output signal from each antenna element is multiplied by a complex

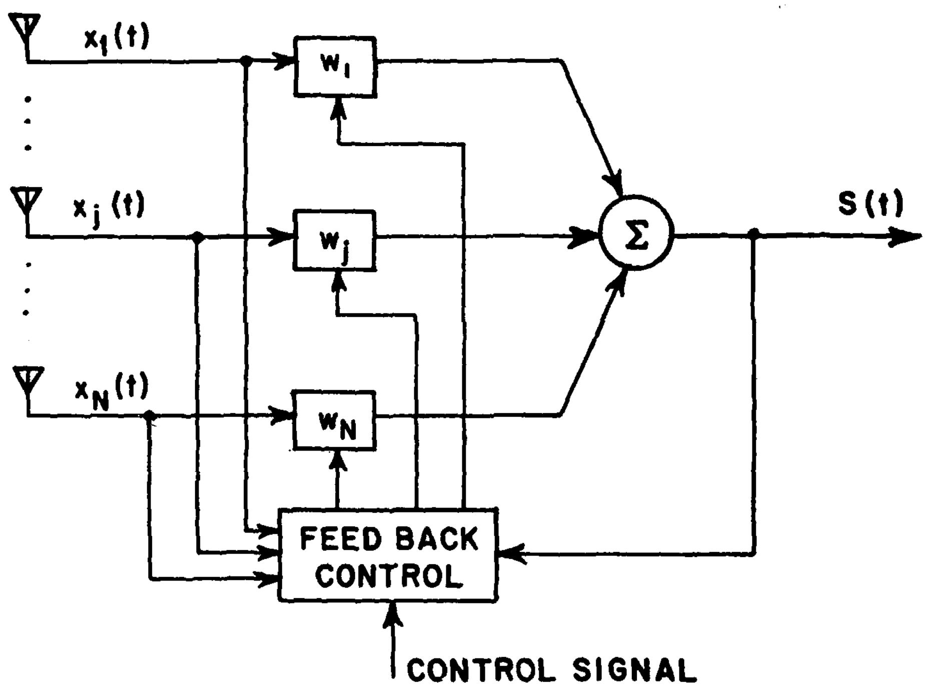

Figure 1. Basic adaptive array.

weight and then these signals are summed to produce the array output $S(t)$. The weights are automatically adjusted to optimize the output SINR in accordance with a selected algorithm. To find an expression for the output SINR, one should know the element output voltages. We will, therefore, first develop an expression for the element output voltages when the mutual coupling is taken into account. These voltages will be used as the input signals to the adaptive processor. The required expression can be obtained by considering the N-element array as an N+1 terminal linear, bilateral network responding to an outside source as shown in Figure 2.

Referring to Figure 2, each port of the N-element array is shown terminated in a known load impedance, Z_L. The array has as its driving source a generator *with open circuit voltage* V_g *and* internal impedance Z_g. Using standard notation, one can write the Kirchoff relation for the N+1 terminal network as

$$\left.\begin{array}{l} v^1 = i_1 Z_{11} + \cdots + i_j Z_{ij} + \cdots + i_N Z_{1N} + i_s Z_{1s} \\ \vdots \\ v^j = i_1 Z_{j1} + \cdots + i_j Z_{jj} + \cdots + i_N Z_{jN} + i_s Z_{js} \\ \vdots \\ v^N = i_1 Z_{N1} + \cdots + i_j Z_{Nj} + \cdots + i_N Z_{NN} + i_s Z_{Ns} \end{array}\right\} \qquad (2.1)$$

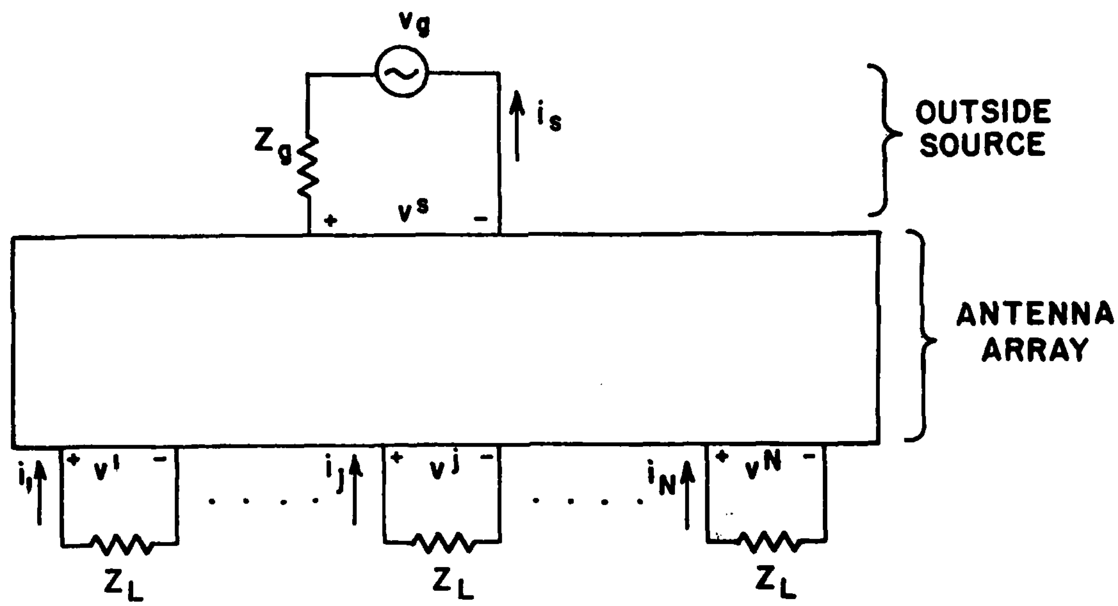

Figure 2. Antenna array as a N+1 terminal network.

Further, making use of the relationship between terminal current and load impedence,

$$i_j = -\frac{v^j}{Z_L}, \quad j = 1, 2, \ldots, N \quad . \tag{2.2}$$

If all the elements in the array are in an open circuit condition then

$$i_j = 0 \qquad j = 1, 2, \ldots, N \quad ,$$

and from Equation (2.1)

$$v^j = v_{0j} = Z_{js} i_s \quad . \tag{2.3}$$

Substituting Equations (2.2) and (2.3) into Equation (2.1) one gets

$$\begin{bmatrix} 1 + \frac{Z_{11}}{Z_L} & \frac{Z_{12}}{Z_L} & \cdots\cdots & \frac{Z_{1N}}{Z_L} \\ \frac{Z_{21}}{Z_L} & 1 + \frac{Z_{22}}{Z_L} & \cdots\cdots & \frac{Z_{2N}}{Z_L} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{Z_{N1}}{Z_L} & \frac{Z_{N2}}{Z_L} & \cdots & 1 + \frac{Z_{NN}}{Z_L} \end{bmatrix} \begin{bmatrix} v^1 \\ v^2 \\ \vdots \\ v^N \end{bmatrix} = \begin{bmatrix} v_{01} \\ v_{02} \\ \vdots \\ v_{0N} \end{bmatrix} \quad . \tag{2.4}$$

Or, more compactly

$$Z_0 V = V_0 \quad . \tag{2.5}$$

In Equation (2.5) Z_0 is the normalized impedance matrix and V_0 represents the open circuit voltages at the antenna terminals. Since Z_0 is nonsingular, one can find the element output voltages from the open circuit voltages. The element output voltages will be given

$$V = Z_0^{-1} V_0 \quad . \tag{2.6}$$

It should be noted that the matrix Z_0 is a normalized impedance matrix, normalized to the load impedance. It acts like a transformation matrix, transforming the open circuit element voltages to the terminal voltages. What is normally assumed in analyzing adaptive antenna systems is that the element spacing is large enough so that the mutual coupling between the elements is small and consequently the matrix Z_0 becomes diagonal. If one further assumes that the self-impedances (Z_{ii}, $i=1,2,...,N$) are equal, the input signal vector will be just the open circuit voltage vector multiplied by a trivial scaling factor involving the self and load impedance terms. Thus the array performance will be the same as calculated using the open circuit voltages as the input signal to an adaptive processor.

Let m+1 CW signals (one desired and m jammers) of the same frequency be incident on the array. Then the open circuit voltages at antenna terminals are given by

$$V_o = X_d + \sum_{k=1}^{m} X_{ik} \tag{2.7}$$

where

$$X_d = A_d \, e^{j(\omega_0 t + \psi_d)} \, U_d \tag{2.8}$$

$$X_{ik} = A_{ik} \, e^{j(\omega_0 t + \psi_{ik})} \, U_{ik} \quad . \tag{2.9}$$

In Equations (2.8) and (2.9), $A_d{}^2$ is the average power in the desired signal, $A_{ik}{}^2$ is the average power in the k^{th} jammer, ω_0 is the carrier frequency, ψ_d is the carrier phase of the desired signal at the coordinate origin, ψ_{ik} is the carrier phase of the k^{th} jammer at the coordinate origin and U_d, U_{ik} are, respectively, the desired signal vector and the k^{th} jammer vector defined as follows:

$$U_d = \begin{bmatrix} f_1(\theta_d, \phi_d, p_d) \, e^{j\rho_{d1}} \\ f_2(\theta_d, \phi_d, p_d) \, e^{j\rho_{d2}} \\ \vdots \\ f_N(\theta_d, \phi_d, p_d) \, e^{j\rho_{dN}} \end{bmatrix} \tag{2.10}$$

where (θ_d, ϕ_d) defines the desired signal direction, p_d is the polarization of the desired signal, $f_j(\theta, \phi, p)$ is the pattern response of the j^{th} element to a signal incident from direction (θ, ϕ) with polarization p and ρd_j is the desired signal phase at the j^{th} element, measured with respect to the coordinate origin.

$$U_{ik} = \begin{bmatrix} f_1(\theta_{ik}, \phi_{ik}, p_{ik})\, e^{j\rho ik_1} \\ f_2(\theta_{ik}, \phi_{ik}, p_{ik})\, e^{j\rho ik_2} \\ \vdots \\ f_N(\theta_{ik}, \phi_{ik}, p_{ik})\, e^{j\rho ik_N} \end{bmatrix} \tag{2.11}$$

where the notation is analogous to that for the desired signal vector.

Using Equations (2.6) and (2.7), the input signal to the adaptive processor will be

$$V = Z_0^{-1}\ (X_d + \sum_{k=1}^{m} X_{ik}) \quad . \tag{2.12}$$

If thermal noise is also added to each element of the array then the total input signal to the processor will be

$$\begin{aligned} X &= V + X_n \\ &= Z_0^{-1}\ [X_d + \sum_{k=1}^{m} X_{ik}] + X_n \end{aligned} \tag{2.13}$$

where X_n is the noise vector defined as

$$X_n = (n_1(t), n_2(t), \ldots, n_N(t))^T \quad . \tag{2.14}$$

In Equation (2.14), T denotes transpose. In the case of an adaptive array, the signal $x_j(t)$ from the j^{th} element is multiplied by a complex weight $w_j(t)$. The signals are then summed to produce the array output. Using the LMS algorithm [2], the steady state weight vector, W, of the array is given by

$$W = \Phi^{-1}S \tag{2.15}$$

where Φ is the covariance matrix

$$\Phi = E\{X^*X^T\} \tag{2.16}$$

and S is the reference correlation vector

$$S = E\{X^*R(t)\} \quad . \tag{2.17}$$

In Equations (2.16) and (2.17), R(t) is the complex reference signal in the adaptive array [2,3], * denotes complex conjugate and $E\{\cdot\}$ denotes expectation.

From Equations (2.13) and (2.16)

$$\Phi = E\ \{[Z_0^{-1}(X_d + \sum_{k=1}^{m} X_{ik}) + X_n]^*\ [Z_0^{-1}(X_d + \sum_{k=1}^{m} X_{ik}) + X_n]^T\ \}$$

$$= (Z_0^{-1})^*E\ \{[(X_d + \sum_{k=1}^{m} X_{ik}) + Z_0X_n]^*\ [X_d + \sum_{k=1}^{m} X_{ik}) + Z_0X_n]^T\}(Z_0^{-1})^T\ .$$

(2.18)

Assuming that the thermal noise voltages from the array elements are gaussian with zero mean and are uncorrelated with each other, and the carrier phases of the narrowband signals are uniformly distributed on $(0, 2\pi)$ and are statistically independent of each other and of the thermal noise voltages, the covariance matrix Φ is given by

$$\Phi = (Z_o^{-1})^* \left[\sigma^2 Z_o^* Z_o^T + \sum_{k=1}^{m} A_{ik}^2 U_{ik}^* U_{ik}^T + A_d^2 U_d^* U_d^T\right](Z_o^{-1})^T \tag{2.19}$$

where σ^2 is the thermal noise power. From Equation (2.19)

$$\Phi = (Z_o^{-1})^* \sigma^2\left[Z_o^* Z_o^T + \sum_{k=1}^{m} \xi_{ik} U_{ik}^* U_{ik}^T + \xi_d U_d^* U_d^T\right] (Z_o^{-1})^T \tag{2.20}$$

where ξ_d is the ratio of the desired signal power to the thermal noise power and ξ_{ik} is the ratio of the k^{th} jammer power to the thermal noise power. Let

$$R_n = Z_o^* Z_o^T + \sum_{k=1}^{m} \xi_{ik} U_{ik}^* U_{ik}^T \tag{2.21}$$

then

$$\Phi = (Z_o^{-1})^* \sigma^2 (R_n + \xi_d U_d^* U_d^T)(Z_o^{-1})^T \quad . \tag{2.22}$$

Note that R_n is the normalized (with respect to the thermal noise power) covariance matrix of the undesired signals (jammer and the thermal noise). To find the steady state weights (Equation (2.15)), Φ^{-1} must be computed. The following matrix inversion Lemma [4], is used to compute Φ^{-1}

$$(A - \alpha Z^* Z^T)^{-1} = A^{-1} - \beta A^{-1} Z^* Z^T A^{-1} \tag{2.23}$$

where A is a nonsingular NxN matrix, Z is a Nx1 column and α, β are scalars related by

$$\alpha^{-1} + \beta^{-1} = Z^T A^{-1} Z^* \quad . \tag{2.24}$$

Using the matrix inversion lemma to invert Φ in Equation (2.22) one gets

$$\Phi^{-1} = \frac{Z_o^T}{\sigma^2} (R_n^{-1} - \tau R_n^{-1} U_d^* U_d^T R_n^{-1})((Z_o^{-1})^*)^{-1} \tag{2.25}$$

where

$$\frac{1}{\tau} = \frac{1}{\xi_d} + U_d^T R_n^{-1} U_d^* \quad . \tag{2.26}$$

The array will acquire and track the desired signal if the reference signal is correlated with the desired signal and is uncorrelated with interference signals. Assuming that the reference signal R(t) is given by,

$$R(t) = A_r e^{j(\omega_0 t + \psi_d)} \tag{2.27}$$

and using Equation (2.13), Equation (2.17) yields

$$S = A_r A_d (Z_o^{-1})^* U_d^* \quad . \tag{2.28}$$

Using Equations (2.25) and (2.28) the steady state weights (Equation (2.15)) of the array are given by

$$W = K Z_o^T R_n^{-1} U_d^* \tag{2.29}$$

where

$$K = \frac{A_r A_d}{\sigma^2} \left(1 - \tau\, U_d^T\, R_n^{-1}\, U_d^*\right) \tag{2.30}$$

is a constant. The weights given in Equation (2.29) will lead to the maximum output SINR in the presence of multiple jammers [see Appendix]. Knowing the steady state weight vector, one can compute the output SINR of the array which is given by

$$\text{SINR} = \frac{P_d}{\sum_{k=1}^{m} P_{ik} + P_n} \tag{2.31}$$

where P_d is the output desired signal power.

$$P_d = \frac{1}{2} E\{|(Z_o^{-1} X_d)^T W|^2\} = \frac{A_d^2}{2} \left|U_d^T\, (Z_o^{-1})^T W\right|^2 \quad . \tag{2.32}$$

P_{ik} is the output interference power due to the k^{th} jammer

$$P_{ik} = \frac{1}{2} E\{|(Z_o^{-1} X_{ik})^T W|^2\}$$

$$= \frac{A_{ik}^2}{2} \left|U_{ik}^T\, (Z_o^{-1})^T W\right|^2 \quad . \tag{2.33}$$

and P_n is the output thermal noise power

$$P_n = \frac{\sigma^2}{2} |W|^2 \quad . \tag{2.34}$$

Using Equations (2.32)-(2.34), Equation (2.31) yields

$$\mathrm{SINR} = \xi_d \, U_d^T \, R_n^{-1} \, U_d^* \quad . \tag{2.35}$$

Equation (2.35) is used to compute the steady state output SINR of an adaptive array consisting of N half-wavelength, center-fed dipoles. All the dipoles are assumed to have similar radiation characteristics and are spaced at a distance d apart (Figure 3). The desired signal and all jammers are assumed to be theta polarized (Figure 3). For the results presented here, $f_k(\theta,\phi,p) = 1.28142$ [Equation (2.10].

Figure 4 shows the output SINR of an adaptive array of six dipoles as a function of the desired signal direction. The dipoles are spaced at a distance of half a wavelength and each dipole is terminated in a load impedance equal to the complex conjugate of the self impedance of a half-wavelength, center-fed dipole. The input signal-to-noise ratio (ξ_d) is 5 dB and the output SINR is computed in the absence of all jammers. The continuous curve in the figure shows the output SINR when the mutual coupling between the array elements is taken into account while the broken curve (--) represents the output SINR when the mutual coupling between the array elements is ignored. Note that the presence of mutual coupling changes the array performance and the output SINR of the array depends on the angle of arrival of the desired signal. The dependence of the array output SINR on the angle of arrival of the desired signal can be explained as follows. Mutual coupling changes the desired signal component of the element output voltages [Equation (2.6)]. The array illumination due to the desired signal is no longer

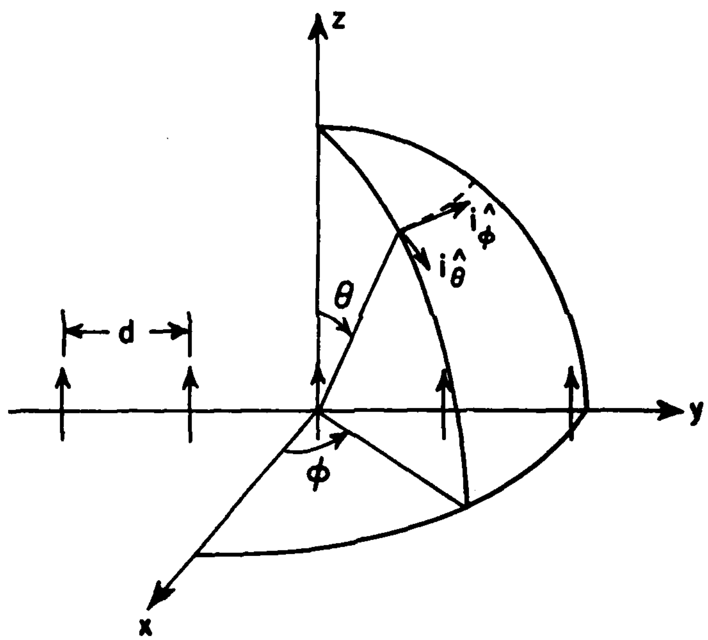

Figure 3. An array of N half-wavelength, center-fed dipoles.

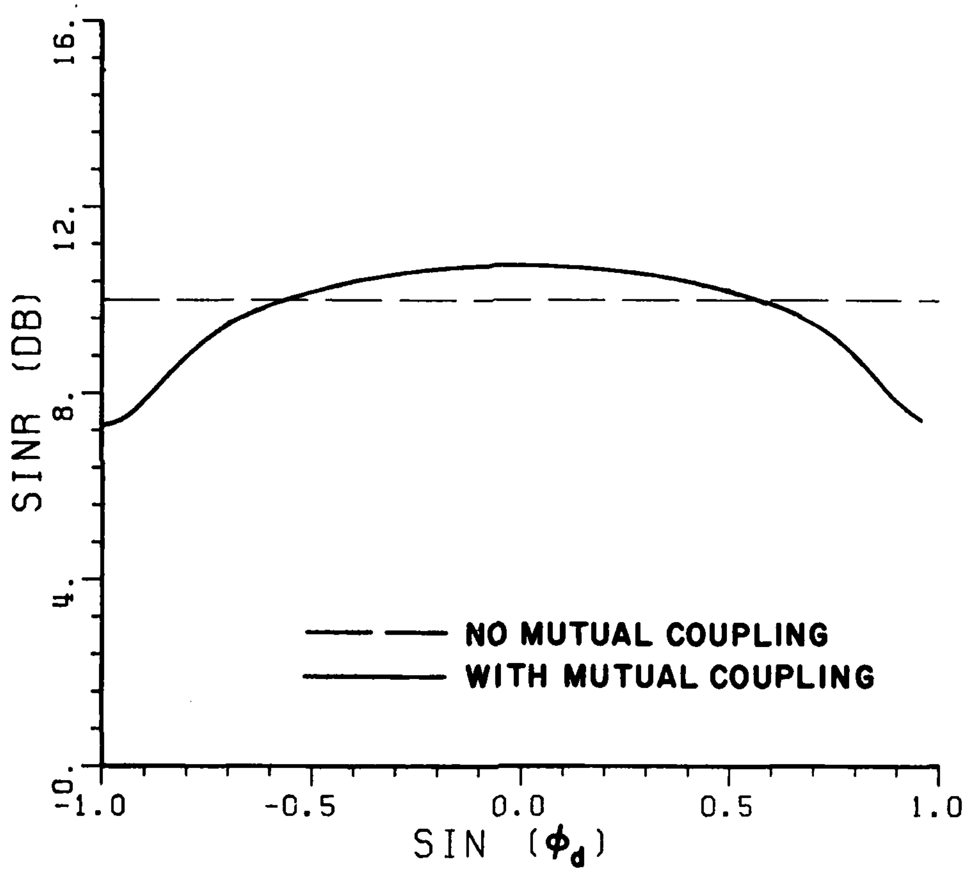

Figure 4. Output SINR of an array of six half-wavelength, center-fed dipoles vs. the desired signal direction (ϕ_d). ξ_d=5 dB, θ_d=90°, d=0.5λ, $Z_L = Z_{ii}^*$.

uniform and depends on the angle of arrival of the desired signal, while the noise being internal is not affected by the mutual coupling between the array elements. The output SINR of the array, therefore, changes with the angle of arrival of the desired signal.

In the above example, the interelement spacing was large ($\lambda/2$) and we found that the mutual coupling does nevertheless affect the performance of the array. For small interelement spacings, the mutual coupling between the array elements will be large (Figure 5) and, therefore, the array performance will be affected more. This is evident in the plots of Figure 6, where the output SINR of the array is plotted as a function of the interelement spacing. The desired signal is incident from the broadside direction (90°,0°) and jammers are assumed to be absent. The broken curve in the plot shows the output SINR in the absence of mutual coupling while the continuous curve shows the output SINR when the mutual coupling is taken into account. Note that the mutual coupling between the array elements affects the array performance even for large interelement spacing ($d > \lambda/2$). The effect is more pronounced for small interelement spacing ($d < \lambda/2$), where the output SINR drops below the expected value (in the absence of mutual coupling) by a significant amount. The change in the output SINR can again be explained using Equation (2.6). In the presence of mutual coupling, the array illumination due to the desired signal deviates from the uniform illumination. The amount of deviation depends upon how strongly the array elements are coupled. The noise being internal to the processors is not affected by the mutual coupling between the array elements. The

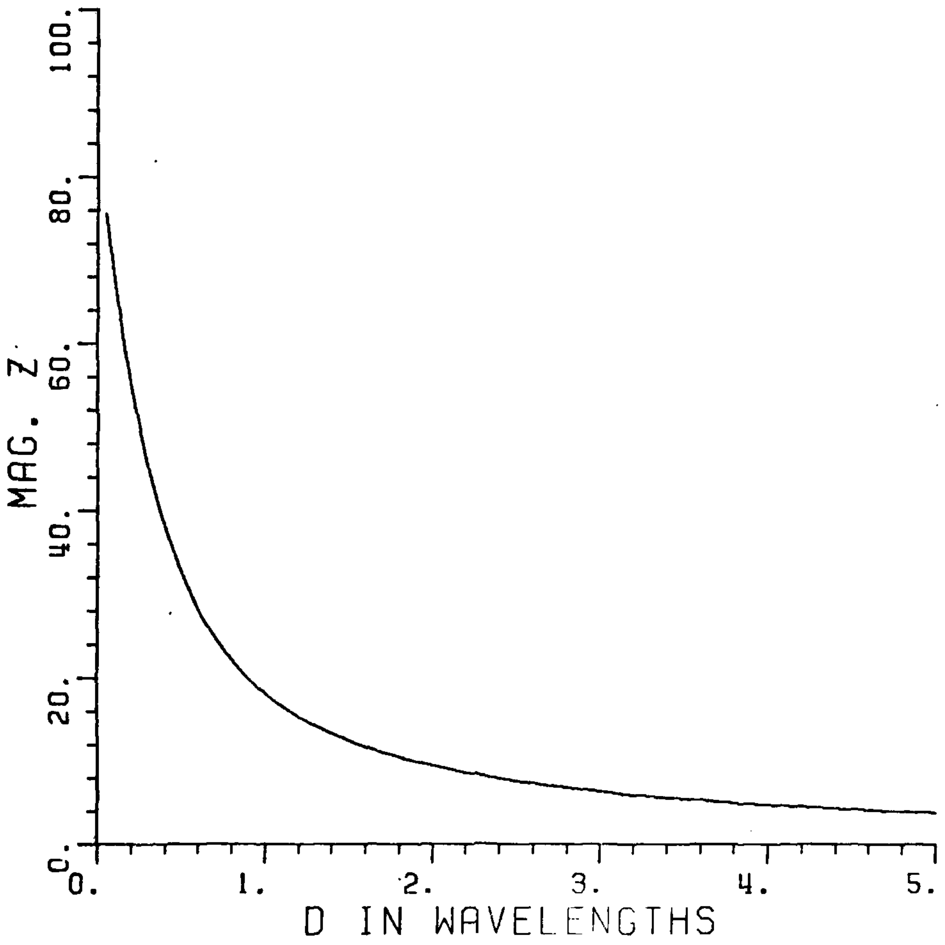

Figure 5. Mutual impedance between two half-wavelength, center-fed thin dipoles vs. the spacing between the dipoles.

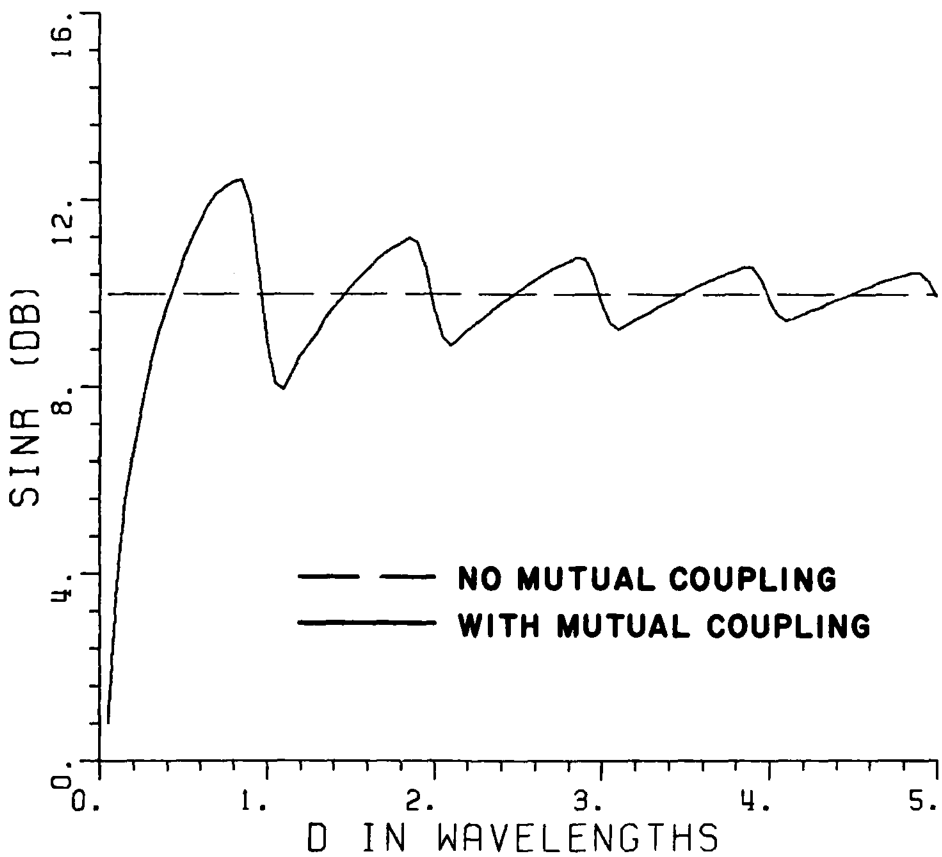

Figure 6. Output SINR of an array of six half-wavelength, center-fed dipoles vs. the interelement spacing. ξ_d=5 dB, $Z_L=Z_{ii}^*$, (θ_d, ϕ_d)=(90°, 0°).

output SINR of the array, therefore, changes with the interelement spacing.

The drop in the output SINR for small interelement spacings can also be related to the reduction of the total incident energy. As the interelement spacing is decreased, the total aperture of the antenna decreases and so does the total incident energy due to the desired signal. Since the receiver internal noise remains unchanged, the signal-to-noise ratio drops. For the same reason, the introduction of additional elements into a fixed aperture with half wavelength spaced antenna elements can degrade the adaptive array performance. The total aperture is fixed and so is the total incident energy. The introduction of additional elements will add to the thermal noise without increasing the available signal power and that will degrade the array output SINR. Figure 7 shows the output SINR of an adaptive array as a function of the number of antenna elements. The array is a linear array of half-wavelength, center-fed dipoles. The total aperture is fixed at 2λ and the desired signal is incident from the broadside direction. Again jammers are absent. The output SINR is computed with and without mutual coupling. In these plots only the indicated points are meaningful (the total number of antenna elements is always an integer). Note that in the absence of mutual coupling the output SINR increases with the introduction of additional elements. It is consistent with the previous work of Compton [5]. But in the presence of mutual coupling, the array output SINR decreases with the introduction of additional elements. One can see that the output SINR

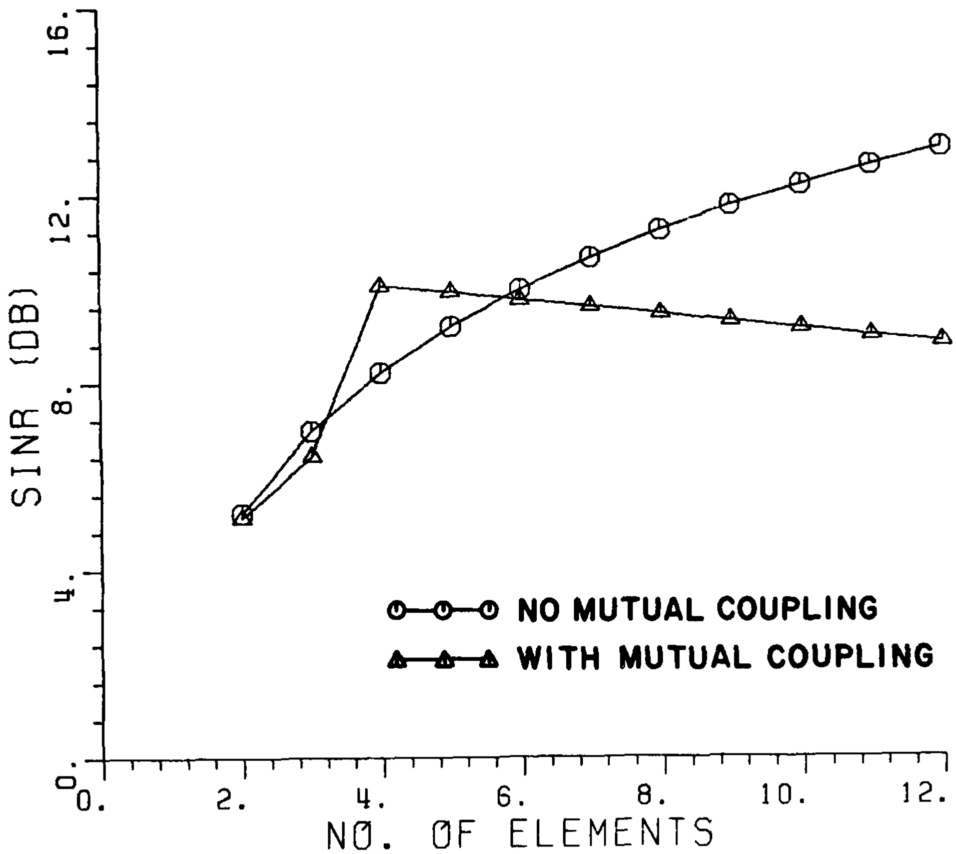

Figure 7. Output SINR of an array of half-wavelength, center-fed dipoles of fixed aperture vs. the number of elements. ξ_d=5 dB, $Z_L=Z_{ii}^*$, $(\theta_d,\phi_d)=(90°, 0°)$, total aperture = 2λ.

reaches a maximum for a four-element array and the array performance degrades with the introduction of additional elements. The array performance improves as the number of antenna elements is increased from two to three to four. The reason for this is that one needs a minimum number of antenna elements to receive all the energy incident on a given aperture. But beyond this point the aperture becomes overcrowded and thus, a densely packed array ($d < \lambda/2$) may lead to a worse performance in an adaptive mode.

In the examples given so far, the array performance was computed in the absence of jammers. The presence of jammers will degrade the array performance. As pointed out in our earlier work [1], the degradation in the array performance can be computed using the unperturbed pattern of the array. The unperturbed pattern of an adaptive array was defined to be proportional to the radiation pattern of the array responding to a single desired signal in the absence of interfering signals. In the absence of jammers, the normalized noise covariance matrix [Equation (2.21)] becomes

$$R_n = Z_o^* Z^T \quad . \tag{2.36}$$

Substituting Equation (2.36) in Equation (2.29), the steady state vector of the array will be

$$W = K(Z_o^{-1} U_d)^* \quad . \tag{2.37}$$

Thus, the weight vector for the unperturbed pattern will be $(Z_o^{-1} U_d)^*$ and the value of the unperturbed pattern in the direction (θ, ϕ) for polarization p will be

$$E(\theta, \phi, p) = (Z_o^{-1} U)^T (Z_o^{-1} U_d)^* \tag{2.38}$$

where U is the signal vector of the array in direction (θ, ϕ) for polarization p. Substituting Equation (2.36) into Equation (2.35), the output SINR of the array in the absence of all jammers will be

$$\begin{aligned} \text{SINR} &= \xi_d \, (Z_o^{-1} U_d)^T \, (Z_o^{-1} U_d)^* \\ &= \xi_d \, E(\theta_d, \phi_d, p_d) \end{aligned} \tag{2.39}$$

and is proportional to the value of the unperturbed pattern (Equation 2.38)) in the desired signal direction. Further, following the same procedure as given in reference [1], it can be shown that the output SINR of the array in the presence of one jammer will be given by

$$\text{SINR} = \xi_d \left[E(\theta_d, \phi_d, p_d) - \frac{|E(\theta_{i1}, \phi_{i1}, p_{i1})|^2}{(Z_o^{-1} U_{i1})^T (Z_o^{-1} U_{i1})^*} \right] \tag{2.40}$$

where $E(\theta_{i1}, \phi_{i1}, p_{i1})$ is the value of the unperturbed pattern in the jammer direction. The same can be done for multiple jammers. Thus, the degradation in the array performance can be computed using the unperturbed pattern.

In this section, the effect of mutual coupling between the array elements on the steady state performance of an LMS type adaptive array was presented. It was shown that though LMS adaptive arrays produce the maximum obtainable output SINR (see Appendix), their performance is affected by mutual coupling. One should, therefore, take mutual coupling into account to compute the true output SINR of the array. In the next section, the transient response of an adaptive array in the presence of mutual coupling will be studied.

III. TRANSIENT RESPONSE OF AN ADAPTIVE ARRAY IN THE PRESENCE OF MUTUAL COUPLING

The speed of response of an adaptive array is controlled by the eigenvalues of its signal covariance matrix [3]. As pointed out in the last section, the presence of mutual coupling between the array elements affects the input signals to the adaptive processor and thus the covariance matrix. The eigenvalues of the covariance matrix will, therefore, be different than those in the absence of mutual coupling. In this section, the transient response of an adaptive array in terms of the eigenvalues of its covariance matrix will be studied. From Equation (2.20), the covariance matrix, Φ, can be written as

$$\Phi = \sigma^2 \left[I + \sum_{k=1}^{m} \xi_{ik} (Z_o^{-1} U_{ik})^* (Z_o^{-1} U_{ik})^T + \xi_d (Z_o^{-1} U_d)^* (Z_o^{-1} U_{ik})^T \right] \cdot \tag{3.1}$$

In the presence of m+1 signals (one desired signal and m jammers), Φ has, at least, N-m-1 eigenvectors (N is the total number of antenna elements) having unity eigenvalues (assuming σ^2=1) and the rest of the eigenvectors have eigenvalues larger than one. The presence of mutual coupling between the array elements will affect these eigenvalues. We will, therefore, compute the nonunity eigenvalues to study the effect of mutual coupling on the transient response of adaptive arrays.

Figure 8 shows the nonunity eigenvalue of a six half-wavelength, center-fed dipole adaptive array in the presence of one signal only (no jammer). The desired signal is incident from the broadside direction (90°, 0°) and is 10 dB stronger than the thermal noise. The eigenvalue is plotted as a function of the interelement spacing. Note that the mutual coupling between the array elements affects the eigenvalues even for large interelement spacing, but the effect is more severe when the spacing is less than half a wavelength. For small spacings, the eigenvalue drops significantly below the value obtained in the absence of mutual coupling (continuous line). The drop in the eigenvalue indicates a reduction in the speed of response of the adaptive array. In other words, the array will take more time to adapt to the changes in the desired signal parameters. The main feature of an adaptive array is nulling the undesired signals (jammers). The transient response of an adaptive array in the presence of jammers, therefore, will be considered next.

Figures 9 and 10 show the nonunity eigenvalues in the presence of one and two jammers respectively. The angles of arrival of the two jammers are (90°,30°) and (90°,-45°) respectively and the jammers are 10 dB and 20 dB stronger than the desired signal. Again the eigenvalues are plotted as a function of the interelement spacing. Note that the

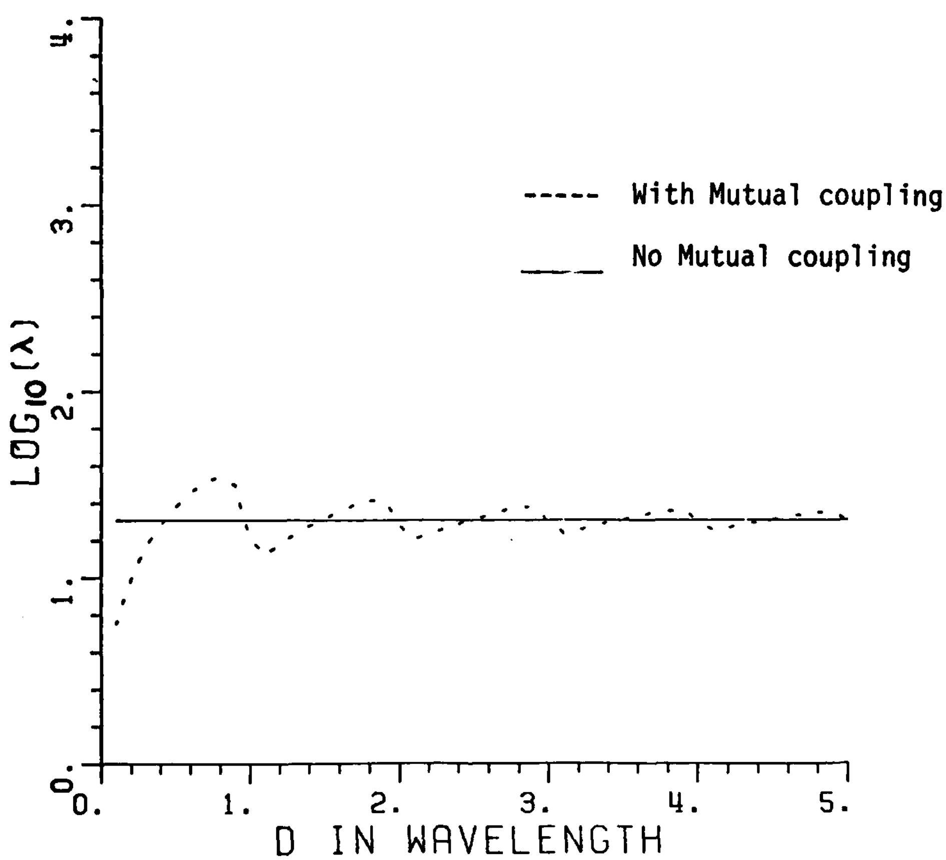

Figure 8. Nonunity eigenvalue (λ) of an array of six half-wavelength, center-fed dipoles in the presence of a desired signal vs. the interelement spacing. ξ_d = 10 dB, $(\theta_d, \phi_d) = (90°, 0°)$ $Z_L = Z_{11}^*$, No Jammer.

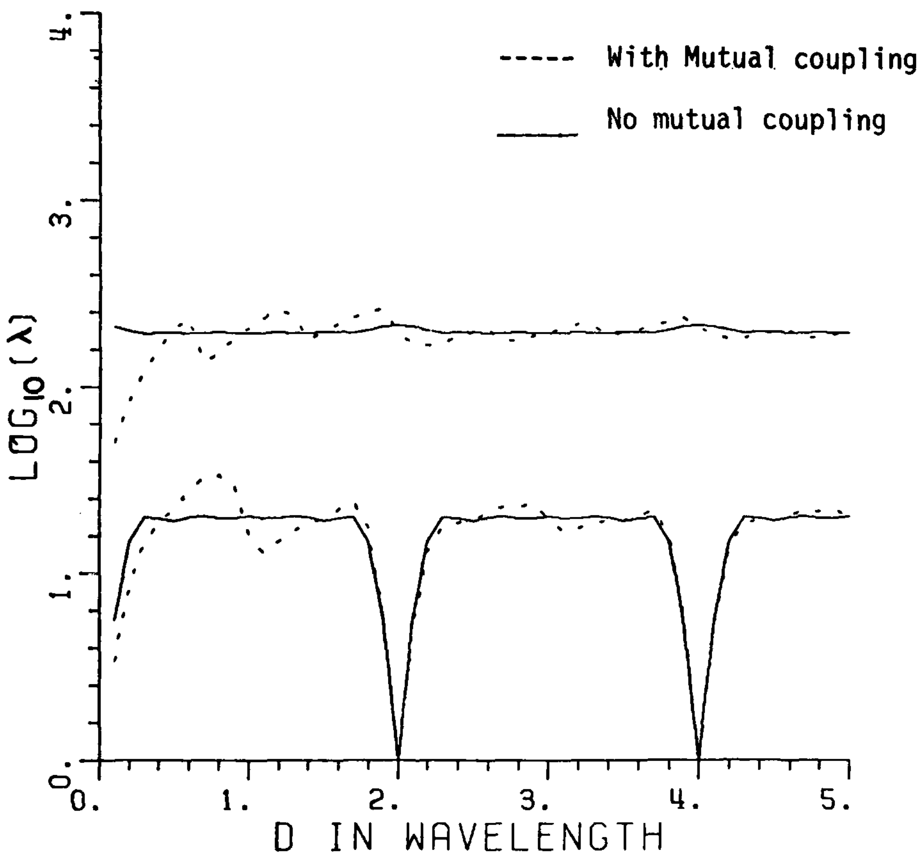

Figure 9. Nonunity eigenvalues (λ) of an array of six half-wavelength, center-fed dipoles in the presence of a desired signal and a jammer vs. the interelement spacing. ξ_d = 10 dB, (θ_d, ϕ_d) = (90°, 0°), ξ_{i1} = 20 dB, (θ_{i1}, ϕ_{i1}) = (90°, 30°), $Z_L = Z_{ii}^*$.

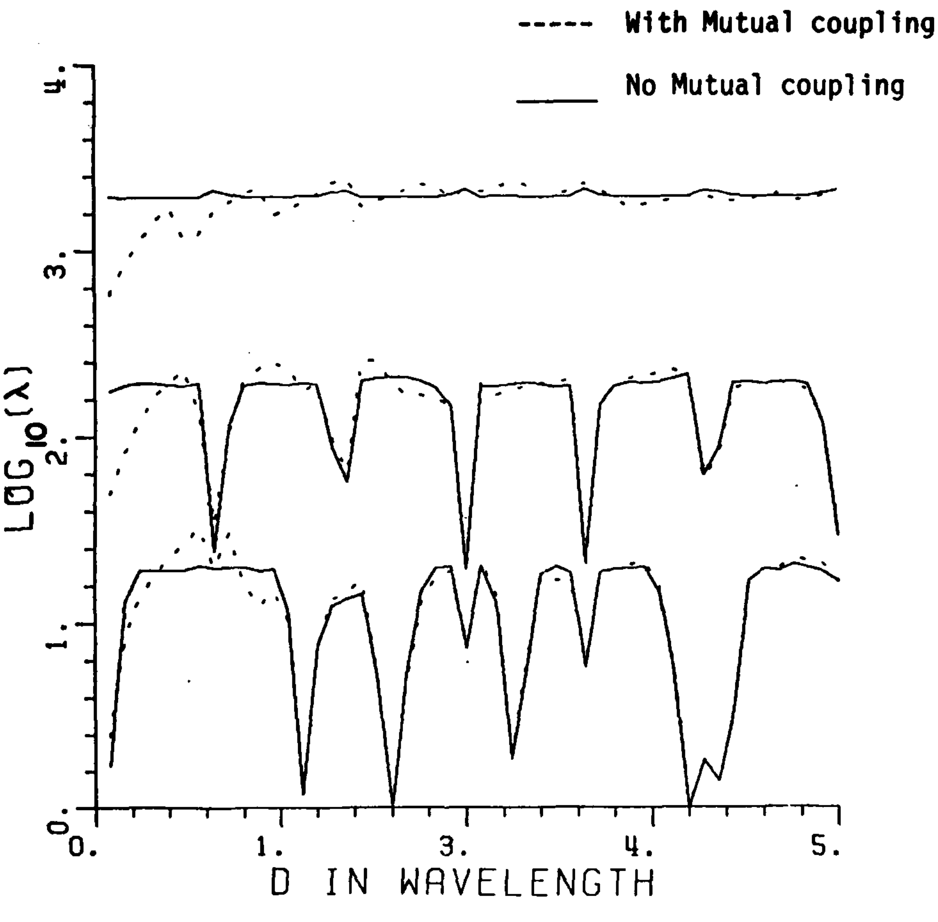

Figure 10. Nonunity eigenvalues (λ) of an array of six half-wavelength, center-fed dipoles in the presence of a desired signal and two jammers vs. the interelement spacing. ξ_d = 10 dB, (θ_d, ϕ_d) = (90°, 0°), ξ_{i1} = 20 dB, ξ_{i2} = 30 dB, (θ_{i1}, ϕ_{i1}) = (90°, 30°), (θ_{i2}, ϕ_{i2}) = (90°, -45°), $Z_L = Z_{ii}^*$.

mutual coupling between the array elements affects the eigenvalues even for large interelement spacings, but the effect is more severe for small interelement spacing ($d \leq \lambda/2$). For such spacing, the eigenvalues drop substantially below the values that they would have had in the absence of mutual coupling (continuous curve). The smaller the eigenvalues, the longer will be the transient, or, the array will take more time to null the jammers, which may be undesirable. The strong fluctuations in the eigenvalues for large interelement spacings are due to the fact that one or more signals (jammer as well as the desired signal) are incident from grating lobe directions.

In this section, the effect of mutual coupling on the transient response of an adaptive array was studied. It was shown that the presence of mutual coupling between the array elements affects the transient behavior of the adaptive array and slows its response to both desired as well as jamming signals for closely spaced elements. In the next section, it is shown that mutual coupling affects the performance of Applebaum type adaptive arrays and that the steering vector for these arrays must be modified to account for the mutual coupling in order to maximize the output SINR.

IV. APPLEBAUM ARRAYS

In the case of an Applebaum adaptive array, one uses a steering vector or initial weights instead of a reference signal as the control signal (Figure 1). The steady state weight vector for this type of adaptive array [6] is given by

$$W = [\frac{1}{G} I + \Phi]^{-1} U_S \tag{4.1}$$

where U_S is the steering vector, G is the loop gain and I is a NxN identity matrix. Note that Equation (4.1) contains the signal covariance matrix, Φ. As pointed out earlier, the presence of mutual coupling between array elements will change Φ and thus will affect the steady state performance of Applebaum type adaptive arrays. Eigenvalues of the signal covariance matrix control the speed of response of Applebaum type adaptive arrays too. The presence of mutual coupling between array elements will, therefore, affect the transient response of Applebaum type adaptive arrays in the same fashion as pointed out in the last section.

Next, we will find the optimum steering vector to maximize the output SINR of Applebaum type adaptive arrays in the presence of mutual coupling. Assuming that the loop gain is large, the steady state weights [Equation (4.1)] becomes

$$W = \Phi^{-1} U_S \quad . \tag{4.2}$$

Using Equation (2.25) in Equation (4.2) one gets

$$W = \frac{Z_o^T}{\sigma 2} (R_n^{-1} - \tau R_n^{-1} U_d^* U_d^T R_n^{-1}) Z_o^* U_S \quad . \tag{4.3}$$

If the steering vector is chosen to steer the beam in the desired signal direction, i.e., $U_S = U_d^*$ then the steady state weight vector will be given by

$$W = \frac{Z_o^T}{\sigma^2} (R_n^{-1} - \tau R_n^{-1} U_d^* U_d^T R_n^{-1}) Z_o^* U_d^* \quad . \tag{4.4}$$

Comparing Equations (2.29) and (4.3) one can see that the two are not the same, and thus this choice of the steering vector would not give the optimum SINR. If instead of the "open circuit" desired signal vector (U_d), the complex conjugate of the desired signal component of the element output voltage is used to generate the steering vector, i.e.,

$$U_S = (Z_o^{-1} U_d)^* \quad , \tag{4.5}$$

then from Equation (4.3)

$$W = K_1 Z_o^T R_n^{-1} U_d^* \quad , \tag{4.6}$$

where

$$K_1 = \frac{1}{\sigma^2} (1 - \tau U_d^T R_n^{-1} U_d^*) \quad . \tag{4.7}$$

Comparing Equations (2.29) and (4.6), one can see that the two weight vectors differ only by a scale factor. The choice of the steering vector as given in Equation (4.5) will, therefore, lead to the optimum performance of the array.

Figure 11 shows the output SINR of an adaptive array of six half-wavelength, center-fed dipoles as a function of the interelement spacing for the two choices of steering vectors. The desired signal is incident

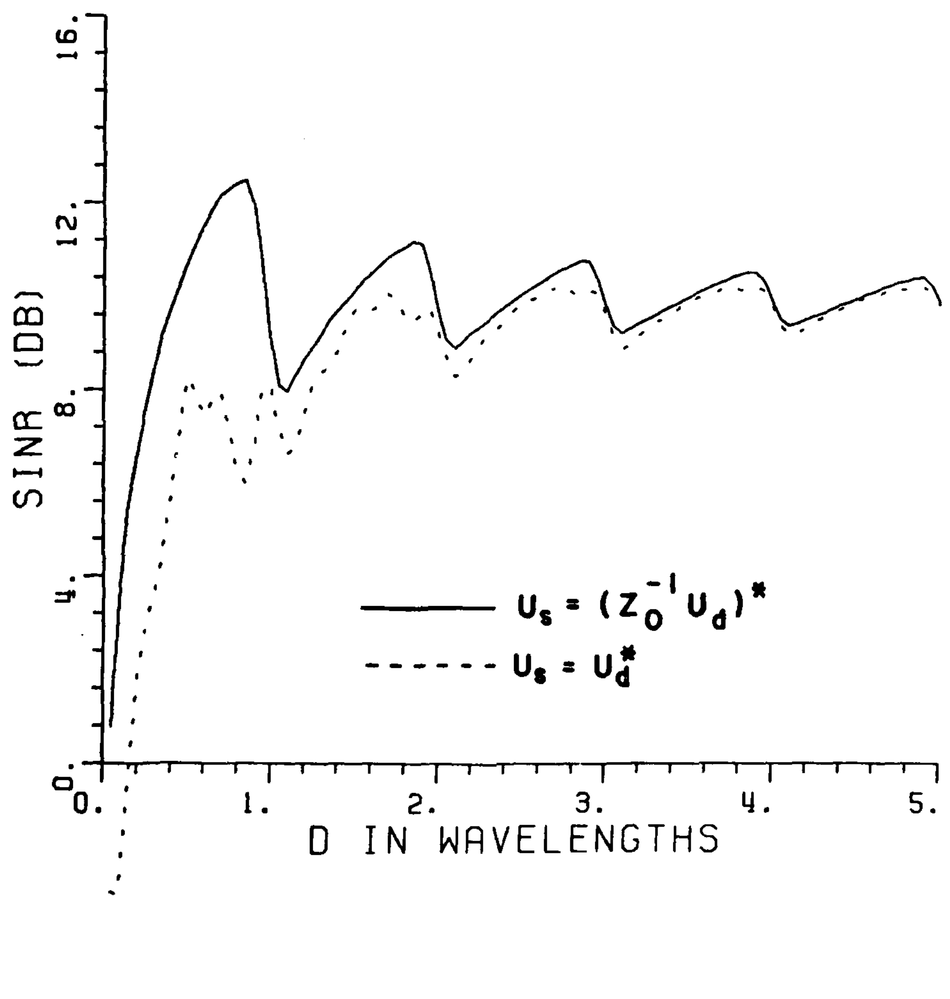

Figure 11. Output SINR of an array of six half-wavelength, center-fed dipoles vs. the interelement spacing using Applebaum algorithm. ξ_d = 5 dB, $Z_L = Z_{ii}^*$, (θ_d, ϕ_d) = (90°, 0°).

from the broadside direction and the load impedance is equal to the complex conjugate of the self impedance of a half-wavelength, center-fed dipole. Again jammers are assumed to be absent. Note that when the steering vector is chosen according to Equation (4.5), the adaptive array gives a better performance for small interelement distances where the mutual coupling is the strongest. Comparing Figures 6 and 11, one can see that the output SINR of the properly excited Applebaum array is the same as that of the LMS array. Thus, $U_S = (Z_0^{-1} U_d)^*$, leads to the optimum performance of an Applebaum type adaptive array.

V. CONCLUSIONS

In this work, the effect of mutual coupling between array elements on the performance of adaptive arrays was studied. It was shown that mutual coupling affects the performance of adaptive arrays even for large interelement spacings. The effect is particularly serious for small interelement spacing where the steady state output SINR of the array is significantly lower than that obtained when mutual coupling is ignored and the speed of response of the array is reduced.

REFERENCES

1. I.J. Gupta and A.A. Ksienski, "Dependence of Adaptive Array Performance on Conventional Array Design", IEEE Trans. on Antennas and Propagation, Vol. AP-30, July 1982, pp. 549-553.

2. B. Widrow, P.E. Mantey, L.J. Griffiths and B.B. Goode, "Adaptive Antenna Systems", Proc. IEEE, Vol. 55, No. 12, December 1967, pp. 2143-2159.

3. R.L. Riegler and R.T. Compton, Jr., "An Adaptive Array for Interference Rejection", Proc. IEEE, Vol. 61, No. 6, June 1973, pp. 748-758.

4. A. S. Householder, The Theory of Matrices in Numerical Analysis, Dover Publication, Inc., New York, 1964, p. 123.

5. R.T. Compton, Jr., "A Method of Choosing Element Patterns in an Adaptive Array", IEEE Trans. on Antennas and Propagation, Vol. AP-30, May 1982, pp. 489-493.

6. S.P. Applebaum, "Adaptive Arrays", IEEE Trans. on Antennas and Propagation, Vol. AP-24, September 1976, pp. 585-599.

APPENDIX A

From Equation (2.21)

$$R_n = Z_o^* \{I + \sum_{k=1}^{m} \xi_{ik} (Z_o^{-1} U_{ik})^* (Z_o^{-1} U_{ik})^T\} Z_o^T . \tag{A.1}$$

Let

$$U_{ik}' = Z_o^{-1} U_{ik} .$$

then Equation (A.1) becomes

$$R_n = Z_o^* \{I + \sum_{k=1}^{m} \xi_{ik} U_{ik}'^* U_{ik}'^T\} Z_o^T .$$

$$= Z_o^* R_n' Z_o^T . \tag{A.2}$$

where

$$R_n' = I + \sum_{k=1}^{m} \xi_{ik} U_{ik}'^* U_{ik}'^T . \tag{A.3}$$

Using Equation (A.2) in Equation (2.29), the steady state weight vector of the array is

$$W = K(R_n')^{-1} (Z_o^{-1} U_d)^*$$

$$= K(R_n')^{-1} U_d'^* \quad . \tag{A.4}$$

where

$$U_d' = Z_o^{-1} U_d \quad . \tag{A.5}$$

Comparing Equation (A.4) with the optimum weight vector [Reference 6, Equation (4.1)] one can see that the two are similar. Thus the weight vector given by Equation (2.29) will lead to the maximum output SINR in the presence of multiple jammers.

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